

# The Mechanics of Metallic Folds

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keywords: origami, folding, metal

## Abstract

The increasing use of origami techniques in engineering applications has necessitated a shift away from paper-based materials to more robust metals and composites. Folding paper locally tears fibres and, with repeated folding, gradually weakens the fold line leading to eventual failure. In contrast, metal folding involves plastic deformation and work-hardening, where yield stresses in the fold robustly rise under repetitive flexing. Metal fold lines exhibit a typically high stiffness, which can subtract from the overall origami character of the structure.

The analysis of origami structures has been dominated by two main techniques. The first assumes rigid plates connected by discrete fold lines. The folds themselves remain straight but accommodate relative rotations across them. Rigid folding this way can be described entirely by a kinematic analysis, even if the fold lines are in some way stiff in practice; adding torsional springs along them does not affect the kinematic description. Typical examples include deployable structures, e.g. solar arrays, if similarly constructed.

The second technique accounts for flexibility beyond the folds. For generality's sake, the fold line is treated as a general space curve connecting thin inextensible surfaces that can only bend. Differential geometry is then used to obtain the strain energy cost of deformation of the facets, as well as the change in fold angle if its elasticity is relevant. This technique has more complexity to solve for, which has prevented its application to large-scale origami structures with multiple folds.

The assumption that an elastic fold can be represented by a well-behaved torsional spring has not been fully investigated. In this work, we investigate the mechanical properties of metallic folds for incorporation into the analysis of origami structures. In contrast to the common singular assumption, we assume a finite geometry for the fold cross-section consisting of a cylindrical segment of radius,  $r$ , sector angle  $\beta$ , and thickness,  $t$ , with linear elastic Young's modulus  $E$ , and Poisson's ratio  $\nu$ . We derive the strain energy per unit length of fold as a function of the change in fold sector angle,  $\theta$ .

In the first instance, we assume that the fold thickness to radius ratio is small enough for thin shell theory to apply. The corresponding strain energy stored per unit length of fold is obtained simply as:  $U = 2D\theta^2/\beta r$ . Where  $r$  is the initial fold radius, and the flexural rigidity  $D = Et^3/12(1 - \nu^2)$ . Such large-radius folds (relative to thickness), however, can result in additional degrees of freedom and more complex behaviour in practice. Metallic folds are commonly

made with a radius of the order of the thickness, where thin shell theory does not apply. Using a solid-mechanics approach, we obtain an equivalent strain energy density for these ‘sharper’ folds.

Both our approaches show that reducing the fold radius increases the energy cost of deformation. In theory, as the fold radius is reduced to zero the stiffness approaches infinity, but in practice this is limited by manufacturing constraints. Due to the large opening resistance of folds with a small radius, the deformation of origami structures made from folded metal will depend to a large extent on deformation of the facets. As a case study we consider a folded metallic strip bent perpendicular to the fold axis and consider the modelling of the fold mechanics as well as the deformation of the facets. The results of this study will improve the understanding of the behaviour of metallic origami structures.